LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**STATISTICS**

THIRD SEMESTER – **APRIL 2019**

ST 3506– MATRIX AND LINEAR ALGEBRA

Date: 25-04-2019 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART-A

Answer ALL the questions. **(10 x 2 =20 marks)**

1. State any two properties of determinants.

2. When is a matrix said to be Skew-Hermitian?

3. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$

4. Define rank of a matrix.

5. When are two vectors said to be equal?

6. What is meant by linear dependence of vectors?

7. State any two properties of linear transformations.

8. Define orthogonal transformation.

9. If X is a characteristic vector of a matrix A, then prove that X cannot correspond to more than one characteristic value of A.

10. Find the characteristic roots of $\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$.

PART –B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. If A and B are two square matrices, find the trace of the sum and product of A and B.

12. Prove that the eigen values of a symmetric matrix are all real and those of a skew symmetric matrix are either purely imaginary or zero.

13. If the non-singular matrix A is symmetric, then prove that A^{-1} is also symmetric.

14. Find the rank of a matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{pmatrix}$

15. Show that the vectors $X_1 = (1, 2, 4)$ and $X_2 = (3, 6, 12)$ are linearly dependent.

16. Prove that every orthogonal set of non-zero vectors is linearly independent.

17. Show that the determinant of a square matrix is equal to the product of its Eigen values.

18. Determine the Eigen values of the matrix A = $\begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$

PART C

Answer any TWO questions. **(2 x 20 = 40 marks)**

19. (a) Prove that every Hermitian matrix A can be written as A = B + iC, where B is real and symmetric and C is real and skew symmetric.

(b) Prove that if any two rows of a determinant are interchanged, then the value of the determinant is multiplied by -1.

20. (a) Find A^{-1} if A = $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

(b) Determine the rank of the matrix: $\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{pmatrix}$

21 (a) Solve the following equations using Cramer's rule.

$x + y + z = 9$; $2x + 5y + 7z = 52$; $2x + y - z = 0$

(b) Show that the vectors $X_1 = (3, 1, -4)$ and $X_2 = (2, 2, -3)$ are linearly independent.

22.(a) State and Prove Cayley- Hamilton theorem.

(b) Verify Cayley-Hamilton Theorem for the following matrix:

$$\begin{pmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{pmatrix}$$

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